NOTES AND CORRESPONDENCE

On the Interpretation of the Diagnostic Quasi-Geostrophic Omega Equation

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ABSTRACT

The usual interpretation of the quasi-geostrophic omega equation can be ambiguous, and an alternative but complementary approach is suggested. In the middle troposphere, upward motion is shown to be the consequence of the cyclonic advection of vorticity by the thermal wind. This relates to several empirical-dynamical rules of synoptic meteorology.

1. Introduction

The vertical velocity in the atmosphere, as derived from various forms of the omega equation, has been widely used in diagnostic studies of atmospheric systems (e.g. Cressman 1961; Goree and Younkin 1966; Krishnamurti 1968a,b; Brodrick and McClain 1969; Downey *et al.* 1973). Vertical motions can be inferred from satellite cloud photographs but it is difficult to readily make use of this information in routine analysis. This difficulty led to the SINAP (Satellite Input to Numerical Analysis and Prediction) studies which utilized the omega equation to interpret the vertical motions in terms of more conventional analysis input data (McClain *et al.*, 1965; Barr *et al.*, 1966; Hayden and Wiin-Nielsen 1968; Nagle and Hayden 1971). The quasi-geostrophic omega equation has proved particularly useful for this and also in diagnostic studies since it can explain several empirical rules of synoptic meteorology founded upon dynamical principles (Phillips, 1963). However, several doubtful approximations and assumptions have been made owing to the ambiguous nature of a widely used interpretation of the quasi-geostrophic omega equation. This note intends to clarify the interpretation and put forward an alternative which provides a further aid to understanding the physical meaning of the quasi-geostrophic vertical motions in the atmosphere and can more readily be applied to the SINAP problem. Nevertheless, it is not a substitute for the solution of a full balance model in diagnostic studies of atmospheric systems, since nongeostrophic terms can be quite important (Krishnamurti, 1968b).

Part of the analysis follows that of Wiin-Nielsen (1959) and results are consistent with the developmental theorems of Sutcliffe (1947).

2. The quasi-geostrophic omega equation

The quasi-geostrophic vertical motion equation is obtained by elimination of the local time derivative between the vorticity and thermodynamic equations and, ignoring diabatic heating and friction, may be written

$$\sigma\nabla^2\omega+f_0^2\frac{\partial^2\omega}{\partial p^2}=f_0\left[\frac{\partial}{\partial p}\mathbf{V}_\psi\cdot\nabla(\zeta+f)-\nabla^2\left(\mathbf{V}_\psi\cdot\nabla\frac{\partial\psi}{\partial p}\right)\right], \quad (1)$$

$$=f_0\left[\frac{\partial}{\partial p}J(\psi,\nabla^2\psi+f)-\nabla^2J\left(\psi,\frac{\partial\psi}{\partial p}\right)\right], \quad (2)$$

$$=f_0[\quad \mathrm{F1} \quad + \quad \mathrm{F2} \quad],$$

where:

p	pressure
ω	vertical motion in p-coordinates $[=dp/dt]$
f_0	constant value of f, the Coriolis parameter
ψ	streamfunction
$\mathbf{V}_\psi$	nondivergent component of velocity $[=\mathbf{k}\times\nabla\psi=(u,v)]$
ζ	vorticity $[=\nabla^2\psi]$
∇	two-dimensional ∇ operator $\left[=\frac{\partial}{\partial x}\mathbf{i}+\frac{\partial}{\partial y}\mathbf{j}\right]$
σ	static stability $\left[=\sigma(p)=-\frac{1}{\rho}\frac{\partial\ln\theta}{\partial p}\right]$
θ	potential temperature
ρ	density
$J(\alpha,\beta)$	Jacobian $\left[=\frac{\partial\alpha}{\partial x}\frac{\partial\beta}{\partial y}-\frac{\partial\beta}{\partial x}\frac{\partial\alpha}{\partial y}\right]$.

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The standard interpretation of this diagnostic equation is as follows. The left-hand side (LHS) is a sort of three-dimensional Laplacian operating on omega and is therefore approximately equivalent to omega multiplied by a negative coefficient. The right-hand side (RHS) is then a forcing function for omega and is usually split into two parts, which we have called F1 and F2. F1 is the vertical derivative of the absolute vorticity advection, so that upward motion occurs east of pressure troughs where cyclonic vorticity advection increases with height. F2 is the Laplacian of the thermal advection, so that upward motion occurs in regions of pronounced warm advection (where warm advection is a maximum).

The problem with this interpretation is in the separation into these two terms, since they are not usually independent because each contains a common cancelling component (see the development later in this section). This has led to inaccurate assumptions and approximations in using the equation. For example, in the SINAP problem applied to the 1000–500 mb layer, F1 is frequently approximated by the vorticity advection at 500 mb by ignoring the 1000 mb contribution, and F2 is either ignored completely or assumed to reinforce F1 (McClain *et al.*, 1965; Nagle and Hayden 1971). Also several diagnostic studies have attempted to determine whether the vorticity advection or thermal advection terms were primarily responsible for vertical motion (e.g., Goree and Younkin 1966; Kirshnamurti 1968b; Brodrick and McClain 1969). This procedure can be confusing since, in many cases, for the middle troposphere, we show that both terms contribute nearly equal amounts to the vertical motion and part of each term cancels.

The RHS terms of the omega equation may be rewritten

$$\left.\begin{aligned} \mathrm{F1} &= J\left(\frac{\partial\psi}{\partial p},\nabla^2\psi\right)+J\left(\frac{\partial\psi}{\partial p},f\right)+J\left(\psi,\frac{\partial}{\partial p}\nabla^2\psi\right)\\ &\quad\ \mathrm{A}\quad +\quad \mathrm{C}\quad +\quad \mathrm{B}\end{aligned}\right\}, \tag{3}$$

$$\left.\begin{aligned} \mathrm{F2} &= -J\left(\nabla^2\psi,\frac{\partial\psi}{\partial p}\right)-J\left(\psi,\nabla^2\frac{\partial\psi}{\partial p}\right)\\ &\quad\ \mathrm{A}\quad -\quad \mathrm{B}\\ &\quad -2\left[J\left(\frac{\partial\psi}{\partial x},\frac{\partial^2\psi}{\partial x\partial p}\right)+J\left(\frac{\partial\psi}{\partial y},\frac{\partial^2\psi}{\partial y\partial p}\right)\right]\\ &\quad -\quad 2\Lambda\left(\psi,\frac{\partial\psi}{\partial p}\right)\end{aligned}\right\}, \tag{4}$$

where

$$\Lambda(\alpha,\beta)=-\frac{\partial^2\alpha}{\partial x\partial y}\left(\frac{\partial^2\beta}{\partial x^2}-\frac{\partial^2\beta}{\partial y^2}\right)+\frac{\partial^2\beta}{\partial x\partial y}\left(\frac{\partial^2\alpha}{\partial x^2}-\frac{\partial^2\alpha}{\partial y^2}\right).$$

In the above

A is the advection of relative vorticity by the thermal wind $\{=J[(\partial\psi/\partial p),\nabla^2\psi]\}$;

C is the advection of the earth's vorticity by the thermal wind $\{=J[(\partial\psi/\partial p),f]\}$; and

B is the advection of thermal vorticity by the wind $\{=J[\psi,(\partial/\partial p)\nabla^2\psi]\}$.

Λ is known as the deformation function (Wiin-Nielsen, 1959) and may be rewritten as

$$\left.\begin{aligned} \Lambda\left(\psi,\frac{\partial\psi}{\partial p}\right) &= J\left(u,\frac{\partial u}{\partial p}\right)+J\left(v,\frac{\partial v}{\partial p}\right)\\ &= \frac{R}{pf_0}\left[J\left(u,\frac{\partial T}{\partial y}\right)-J\left(v,\frac{\partial T}{\partial x}\right)\right]\\ &= \frac{1}{2}\left[\mathrm{E}\frac{\partial D}{\partial p}-\mathrm{D}\frac{\partial E}{\partial p}\right]\end{aligned}\right\}, \tag{5}$$

where T is the temperature, and $\mathrm{D}=(\partial v/\partial x)+(\partial u/\partial y)$, $\mathrm{E}=(\partial u/\partial x)-(\partial v/\partial y)$ are the deformation expressions for the wind. Therefore, Λ requires horizontal as well as vertical shear in the wind in order to be nonzero. Wiin-Nielsen (1959) discussed this term at some length and makes the following points for the 500 mb level: 1) the term vanishes if ψ is proportional to $\partial\psi/\partial p$, i.e., the wind is parallel to the thermal wind; 2) it is locally smaller in magnitude than term A, and a mean value of Λ turns out to be less than half the corresponding mean value of term A; and 3) the deformation function is generally on a smaller scale than term A. However, he suggests that 2) and 3) do not hold in general for levels above or below 500 mb, where there is greater baroclinicity in the atmosphere. Calculations by the present author in several case studies (see the Appendix) confirm the validity of the above points but indicate the relative smallness of Λ compared to A applies more generally to the middle troposphere (600 and 400 mb).

Term C is also small. It may be written

$$\mathrm{C}=\frac{\partial v}{\partial p}\beta, \tag{6}$$

where $\beta=df/dy$, and it contributes to upward motion if the thermal wind is directed equatorward. However term B has the same order of magnitude as term A.

From (3) and (4), the forcing function on the RHS of (2) becomes

$$\mathrm{F1}+\mathrm{F2}=2\mathrm{A}+\mathrm{C}-2\Lambda\left(\psi,\frac{\partial\psi}{\partial p}\right) \tag{7}$$

$$=J\left(\frac{\partial\psi}{\partial p},2\nabla^2\psi+f\right)-2\Lambda\left(\psi,\frac{\partial\psi}{\partial p}\right). \tag{8}$$

For some purposes and, in particular, for relating to empirical rules of synoptic meteorology, this may be simplified to

$$\left.\begin{aligned} &\mathrm{F1}+\mathrm{F2}\approx 2\mathrm{A}=2J\left(\frac{\partial\psi}{\partial p},\nabla^2\psi\right) && (9a)\\ &\text{or alternatively}\\ &\mathrm{F1}+\mathrm{F2}\approx 2(\mathrm{A}+\mathrm{C})=2J\left(\frac{\partial\psi}{\partial p},\nabla^2\psi+f\right) && (9b)\end{aligned}\right\}. \quad (9)$$

The approximations involved in using (9a) or (9b) are the same so that the interpretation may be applied to either relative or absolute vorticity, which we shall refer to simply as vorticity. Although the approximation is fairly crude, it is very useful for estimating areas of upward or downward motion visually from a chart containing geopotential height and thickness contours, since *upward motion occurs where there is cyclonic advection of vorticity*[2] *by the thermal wind.* Only when the thermal wind is roughly parallel to the contours is this qualitatively similar to the cyclonic advection of vorticity by the wind, but it is clear from (9) that both F1 and F2 contribute equally to the forcing of omega, so that a factor of 2 is included. However, term B, which can be a fairly large part of F1 and F2, cancels.

3. Discussion

In order to reduce the RHS of the quasi-geostrophic omega equation to (9) several approximations were made. However, the interpretation of the LHS of (1) as omega multiplied by a negative coefficient also involves approximations which should be considered as well in assessing the validity of the above "rule" for determining areas of upward or downward motion. The main assumption on the RHS of (8) is the relative smallness of A, which seems to apply fairly well in the middle troposphere but is less accurate below 700 mb or above 350 mb where baroclinicity plays a greater role. Fortunately, this is compatible with the assumptions on the LHS. Once the forcing function F1+F2 has been determined, the LHS of the omega equation may be inverted to determine an exact solution of omega. Experience with several case studies (e.g., see the Appendix) shows that it may be interpreted as omega multiplied by a negative coefficient mainly in the middle troposphere (e.g., 600 and 400 mb) but at other levels the assumptions involving the vertical derivative term are slightly less valid. Therefore, all assumptions appear to be a reasonable but rough approximation in the middle troposphere.

Several diagnostic case studies of different situations have been made in which separate calculations were made of F1 and F2, as in (3) and (4). One case is illustrated in the Appendix. These confirm that F1 and F2 have a fairly large contribution which cancels and makes it difficult to determine the net areas of upward motion visually from a chart when using the more traditional interpretation of the omega equation. Also, any assessment as to which of the F1 or F2 terms gives a more important contribution to omega can be misinterpreted. What in fact is then being done is a determination of whether term B has the same or opposite sign as term A, and is primarily a qualification of term B rather than omega. It may be justified where B is nearly equal to A in magnitude, so that either F1 or F2 nearly vanishes, but becomes misleading if extended to more complicated cases.

Term B is the advection of thermal vorticity by the wind and will be unimportant in the case of a fairly uniform westerly thermal wind, but term A may still be important and equal contributions to omega will come from each of F1 and F2. This appears to be common in the early stages of cyclogenesis, as found in several examples cited by Krishnamurti (1968b). In the case where there is significant curvature in the thickness field, then B will be positive between the cold trough and the downstream warm ridge. Thus B contributes to upward motion from F1 (owing to cyclonic advection of thermal vorticity) but downward motion from F2 (owing to cold advection) which exactly cancel.

Cases where F1 is small compared to F2 are not uncommon. Krishnamurti (1968b) found that total vertical motion was largely a thermal contribution for a major storm in the occlusion stage, so that $\mathrm{A}\approx-\mathrm{B}$. Effects from vorticity advection were present, but small compared to thermal advection. This also occurs when a tropical storm approaches the baroclinic westerlies so that low-level thermal advection becomes marked but is largely offset by vertical motions, and upper level vorticity advection is weak (Trenberth,

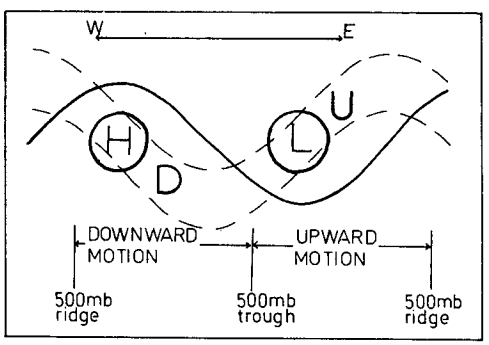

FIG. 1. Schematic 1000 mb contours (solid lines) and 1000–500 mb thickness contours (dashed lines) which indicate regions of vertical motion due to advection of vorticity by the thermal wind. The regions of maximum vertical velocities are indicated by U and D.

[2] Cyclonic advection of vorticity occurs where the relative vorticity is changing to become more cyclonic through advection of vorticity. It is synonymous with positive vorticity advection in the Northern Hemisphere, but the latter term discriminates against the Southern Hemisphere, where the normal convention is to assign negative values to cyclonic vorticity.

1977). In contrast, if the surface pressure field is featureless, and the thermal trough is in phase with the upper level pressure trough, then B will enhance A in its contribution to F1, but will cancel the F2 contribution. An example is the comma-shaped cloud associated with a secondary mid-tropospheric cyclonic vorticity maximum. However, in general, the cancellation of term B that arises in F1 and F2 does not allow such simple conclusions to be made.

On the other hand, the use of (9) is unambiguous since vertical motions are related to the advection of the mean vorticity of a layer by the thermal wind. A schematic representation of this is given in Fig. 1. The most commonly available chart to which this theory may be applied is that representing the 1000 mb height and the 1000–500 mb thickness fields. Assuming that the thickness field is qualitatively similar at other levels, it may be used in conjunction with an assessment of the mid-tropospheric vorticity, either direct from the 500 mb analysis or by adding the thickness and 1000 mb vorticity. Thus upward motion occurs downstream *in the thickness field* from the cyclonic vorticity maximum. In Fig. 1 maximum upward and downward motion are shown by the U and D.

In practice, this rule has been very useful as an aid to understanding the vertical motions in the atmosphere and it can be subjectively used in the SINAP problem. A student who analyses a set of charts manually, using satellite pictures as a guide, should make sure that the analyses are consistent with the rule so that areas of upward motion are also areas where there is cyclonic advection of vorticity by the thermal wind.

The expression (9) is also a justification for the rule that the thickness field is more appropriate for steering short-wave systems and, as such, helps to account for the slowing down and poleward movement of depressions in the maturing stage of a cyclone's development. In the early stage of a developing cyclone where there is a marked westward slope with height, it explains why the upward motion is over the surface low, whereas in later stages of development the upward motion is carried out to the east. These development and steering concepts were originally stated by Sutcliffe (1947), based on dynamical reasoning.

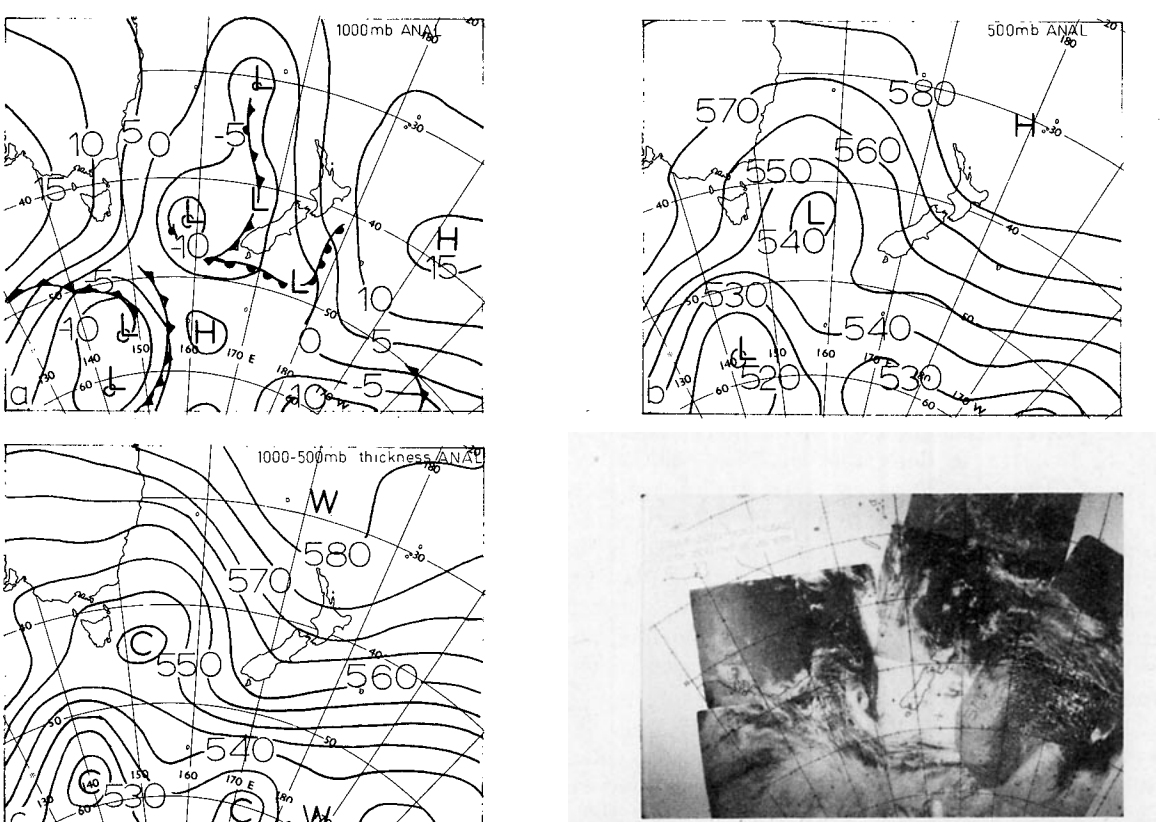

FIG. A1. (a) 1000 mb height analysis (dam) valid 0000 GMT 26 January 1976; (b) 500 mb height analysis (dam) valid 0000 GMT 26 January 1976; (c) 1000–500 mb thickness analysis (dam) valid 0000 GMT 26 January 1976; and (d) satellite cloud mosaic (visible) from ESSA 8 valid about 21 GMT 25 January 1976.

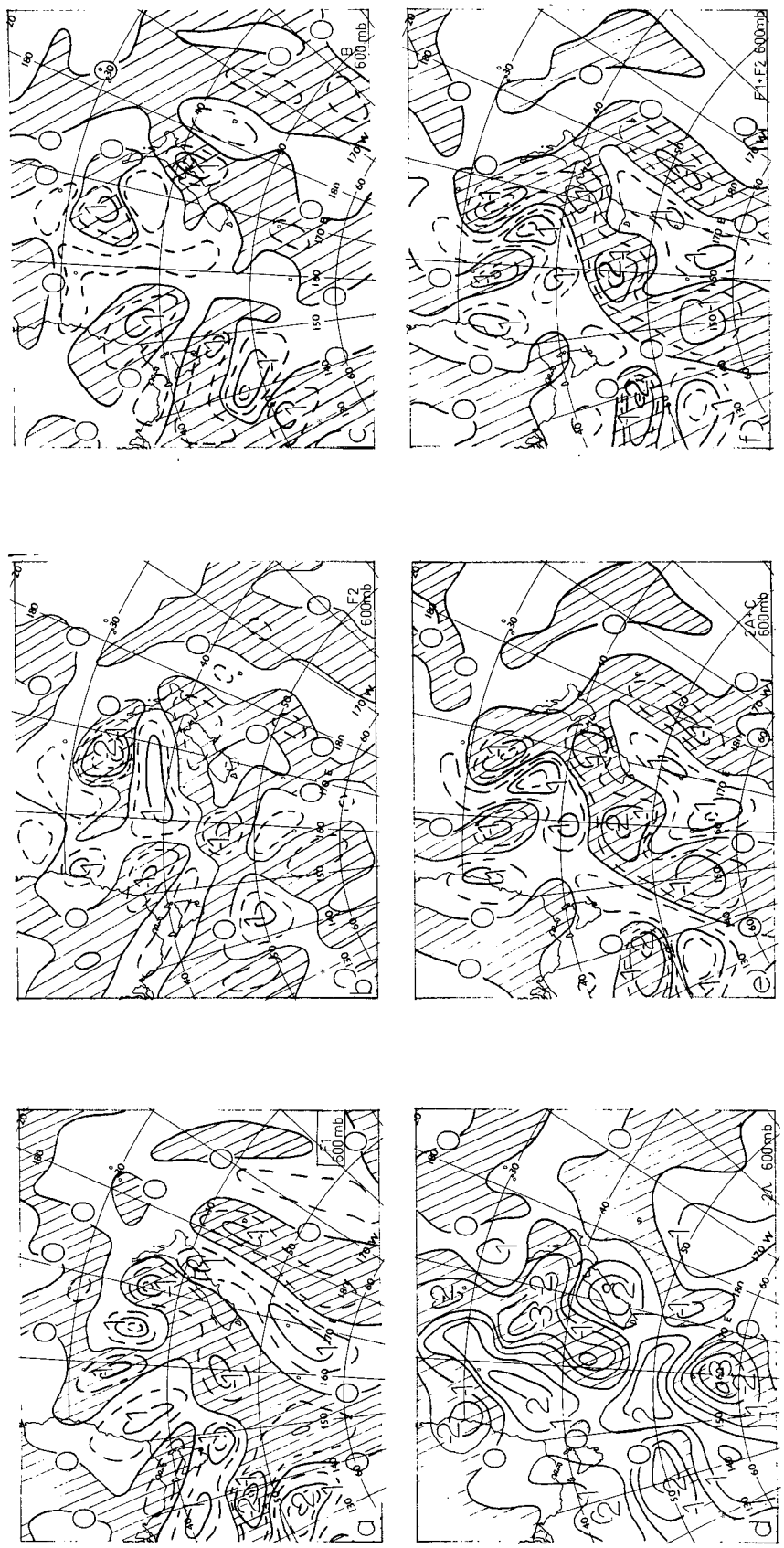

FIG. A2. Terms from the omega equation for 600 mb: (a) F1 (10^{-11} mb^{-1} s^{-2}), (b) F2 (10^{-11} mb^{-1} s^{-2}), (c) B (10^{-11} mb^{-1} s^{-2}), (d) $-2A$ (10^{-12} mb^{-1} s^{-2}), (e) 2A+C (10^{-11} mb^{-1} s^{-2}), (f) F1+F2 (10^{-11} mb^{-1} s^{-2}), (g) ω (10^{-3} mb s^{-1}). See text for further explanation.

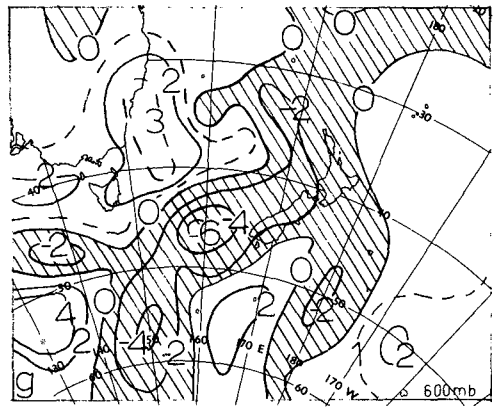

FIG. A2. (*continued*)

4. Conclusions

The more traditional approach of separating the RHS of the quasi-geostrophic omega equation into the two parts—1) the vertical derivative of vorticity advection, and 2) the Laplacian of the thermal advection—can be misleading because they are not independent. In many cases both terms contribute roughly equal amounts to vertical motions in the middle troposphere and part of each term cancels. An alternative but complementary approach has been proposed which removes the ambiguity of this interpretation.

The quasi-geostrophic omega equation has been reanalyzed into a form which readily allows vertical motions to be qualitatively assessed directly from a chart that features geopotential height and thickness contours. Alternatively, the analyses should be constructed so that areas of upward motion, as determined from satellite pictures, are consistent with the rule that upward motion is present where there is cyclonic vorticity advection by the thermal wind. This rule is reasonably valid in the middle troposphere (from about 700 to 350 mb) and also explains several other empirical-dynamical rules of synoptic meteorology. It provides some justification for the concept of steering by the thermal field and helps explain the changing relationship between the main area of upward motion and the center of the depression at the various stages of a cyclone's development.

APPENDIX

Case Study

In order to illustrate the concepts outlined in the paper, detailed calculations of the terms of the omega equation for one case study are presented. The calculations were performed using a grid of 381 km at 60°S on a polar stereographic projection based on height analyses at 1000, 850, 700, 500, 300, and 200 mb. The case shown is for 0000 GMT 26 January 1976 (summer) since analyses and prognoses, using a quasi-geostrophic model, have been presented in detail elsewhere (Trenberth and Neale, 1976, 1977).

FIG. A1 shows analyses for (a) 1000 mb, (b) 500 mb (c) 1000–500 mb thickness and (d) the satellite picture mosaic. An extratropical depression in the south Tasman Sea was undergoing explosive cyclogenesis and moving southeastward. In the north Tasman Sea a decaying tropical cyclone was moving rapidly southeastward to become absorbed in the frontal system of the depression and, together with a secondary low developing west of New Zealand, brought large pressure falls to the region east of New Zealand on 27 January. These analyses show that regions of pronounced thermal and vorticity advection are present and ageostrophic effects are probably significant.

Fig. A2 shows terms of the omega equation for the 600 mb level as (a) F1, (b) F2, (c) B, (d) -2Λ, (e) 2A+C, (f) F1+F2 and (g) ω. For all terms, except B, the negative area is hatched and indicates regions which contribute to upward motion [the f_0 factor in (2) is negative in the Southern Hemisphere]. The static stability value used at 600 mb was 2.1×10^{-4} m^3 mb^{-1} kg^{-1}. Note that the units of Λ are a factor of 10 less than for the other terms.

From (6) the maximum magnitude of term C, corresponding to a 1000–500 mb shear of 45 m s^{-1}, is 1.5×10^{-12} mb^{-1} s^{-2}. Therefore, C is small, A is very similar to $\frac{1}{2}$(2A+C) and 2A+C is very similar to the expressions in (9).

Terms F1, F2, B and A are revealed to be of the same order of magnitude, and the influence of B on both F1 and F2 is evident. Neither F1 nor F2 resemble their sum F1+F2 very closely, whereas the approximation 2A+C in (7) and (9) shows a very strong resemblance. Thus the -2Λ term in (7) is much smaller than 2A+C at 600 mb (by a factor of 5–10). At 775 mb (not shown), it is of greater significance, although still relatively small (by a factor of 3–6).

The similarity between F1+F2 and ω is apparent, but ω is influenced by F1+F2 at other levels and the resemblance between them is usually greatest in the mid-troposphere, where ω is a maximum. A comparison of the ω pattern with the satellite cloud imagery (Fig. A1d) also shows good agreement.

These results are typical of the five other cases which have been similarly analyzed.

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The Expanded North Circumpolar Vortex of 1976 and Winter of 1976–77, and Attendant Vortex Displacement

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ABSTRACT

Planimetering of the area poleward of contours in the main belt of westerlies on 300 mb mean-monthly polar stereographic maps indicates that the size of the 300 mb north circumpolar vortex was larger in 1976 than in any year since initiation of the record in 1963, and that the size of the winter vortex of 1976–77 was also the largest of record. Other "firsts" for the winter of 1976–77 include the displacement of the vortex furthest in the direction of the Greenwich meridian and the closest approach of the vortex center to the North Pole (smallest eccentricity). It is shown that the large vortex size and small eccentricity in the winter of 1976–77 are qualitatively, but not quantitatively, in agreement with the previously noted relations of these parameters with the quasi-biennial wind oscillation of the low tropical stratosphere.

1. Introduction

The areal extent of the north circumpolar vortex at 300 mb has been estimated by planimetering the area north of contours in the main belt of westerlies (the 9280 m contour in spring and fall, the 9120 m contour in winter, and the 9440 m contour in summer) on the mean-monthly polar stereographic maps analyzed since 1963 by the Free University of Berlin, while the center of the vortex has been estimated along axes 90°W–90°E and 0°–180° by finding the two axial locations which divided the total vortex area into two equal areas (Angell and Korshover, 1977a). The total vortex area has been determined to within 1% by repeated planimetering, and application of the same 1% criterion to the planimetering by quadrant leads to an uncertainty in vortex-center location along each axis of ~100 km. In spring and summer the mean vortex-center location is very close to the North Pole, but in winter it is about 500 km from the Pole along 120°E and in fall about 300 km from the Pole along 150°E.

Given the usual contour spacing within the main belt of westerlies, a 1% change in 300 mb vortex area corresponds to a change in mean temperature for the surface —300 mb layer in mid-latitudes of ~0.20°C in winter, 0.10°C in summer and 0.15°C in spring and fall, assuming no change in mid-latitude zonally averaged surface pressure. A change in zonally averaged surface pressure of 0.4 mb would also lead to about a 1% change in vortex area, but the vortex area is much more closely attuned to mid-latitude tropospheric temperature than to surface pressure based on the respective year-average correlations of −0.55 and −0.21 (the latter hardly significant) between these parameters, as determined from the 12 radiosonde stations in the north temperature latitude network (Angell and Korshover, 1977b). Consequently, to a first approximation it can be assumed that the mid-latitude tropospheric temperature change is only slightly less than that indicated above for a given vortex-area change. Note that in the following the pronounced annual variation